CBCS SCHEME

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18AI56

Fifth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Mathematics for Machine Learning

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Determine the values of k such that the system of linear equations $x + y + z = 1$, $x + 5y + 4z = k$ and $x + 4y + 10z = k^2$ is consistent and hence solve. **(07 Marks)**

b. Let W be subspace of R^5 spanned by the vectors

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \\ -1 \end{bmatrix} \quad x_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \\ -2 \end{bmatrix} \quad x_3 = \begin{bmatrix} 3 \\ -4 \\ 3 \\ 5 \\ -3 \end{bmatrix} \quad x_4 = \begin{bmatrix} -1 \\ 8 \\ -5 \\ -6 \\ 1 \end{bmatrix}$$

Find the subset that form the basis for W. **(07 Marks)**

c. (i) Compute the distance between $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $y = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

(ii) Compute the angle between $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $y = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ **(06 Marks)**

OR

2 a. Investigate the values of λ and μ such that the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ may have
(i) Unique solution (ii) Infinite solution (iii) No solution. **(07 Marks)**

b. Find the co-ordinate vector of (10, 5, 0) relative to the vectors (1, −1, 1), (0, 1, 2) and (3, 0, −1). **(07 Marks)**

c. Show that $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in R^2 defined by $\langle u, v\rangle = 3u_1v_1 + 2u_2v_2$ is an inner product space. **(06 Marks)**

Module-2

3 a. Orthogonally diagonalize the matrix

$$A = \begin{bmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$ **(10 Marks)**

b. Find singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$ **(10 Marks)**

OR

4 a. Apply Gram Schmidt orthogonalization process to the basis B = {(1, 0, 1), (1, 0, –1), (0, 3, 4) } of the inner product space R^3 to find an orthogonal basis of R^3. Also find orthonormal basis of R^3. **(10 Marks)**

b. Find Eigen decomposition of the matrix

$$A = \begin{bmatrix} 11 & -4 & 7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$$

(10 Marks)

Module-3

5 a. Compute the Taylor polynomials T_n, for n = 0, 1, 5, 10 for f(x) = sin x + cox x at $x_0 = 0$. **(07 Marks)**

b. Compute the derivative of the function $h(x) = (2x + 1)^4$ using the chain rule. **(06 Marks)**

c. Consider the matrix $R \in \mathbb{R}^{M\times N}$ and $f : \mathbb{R}^{M\times N} \to \mathbb{R}^{N\times N}$ with $f(R) = R^T R = K \in R^{N\times N}$. Find gradient dK/dR. **(07 Marks)**

OR

6 a. Find the gradient df/dx for the function $f(x_1, x_2) = x_1^2 x_2 + x_1 x_2^3 \in R$. **(06 Marks)**

b. Consider the function $h : R \to \mathbb{R}$,

$$h(t) = (f o g)(t) \quad \text{with} \quad f : R^2 \to R, \quad g : R \to R^2, \quad f(x) = e^{x_1 x_2^2}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad g(t) = \begin{bmatrix} t\cos t \\ t\sin t \end{bmatrix}$$

Compute the gradient of h with respect to t. **(07 Marks)**

c. Consider the functions

$f_1(x) = \sin(x_1)\cos(x_2)$, $x \in R^2$

$f_2(x, y) = x^T y$, $x, y \in R^n$

$f_3(x) = xx^T$, $x \in R^n$

(i) What are the dimensions of $\frac{\partial f_i}{\partial x}$? (ii) Compute the Jacobians. **(07 Marks)**

Module-4

7 a. A box A contains two white and four black marbles. Another box B contains five white and seven black marbles. A marble is transferred from box A to box B, then a marble is drawn from B. Find the probability that it is white. **(06 Marks)**

b. Consider the following bivariate distribution p(x, y) of two discrete random variables X and Y. Compute (i) The marginal distributions p(x) and p(y) (ii) The conditional distribution $p(x \mid Y = y_1)$ and $p(y \mid X = x_3)$

Y \ X	x_1	x_2	x_3	x_4	x_5
y_1	0.01	0.02	0.03	0.1	0.1
y_2	0.05	0.1	0.05	0.07	0.2
y_3	0.1	0.05	0.03	0.05	0.04

(07 Marks)

c. Let X be a continuous random variable with probability density function on $0 \le x \le 1$ $f(x) = 3x^2$. Find the probability density function of $Y = X^2$. **(07 Marks)**

OR

8 a. Three machines A, B, C produces 50%, 30% and 20% of the items in a factory. The percentage of defective items are 3%, 4% and 5% respectively. If an item is selected at random, what is the probability that it is defective what is the probability that it is from A. **(06 Marks)**

b. The life of a bulb is a normal variate with a mean life of 2040 hours and standard deviation of 60 hours. In a consignment of 2000 lamps, find how many would be expected to burn for (i) more than 2150 hours (ii) less than 1950 hours, (iii) between 1920 hours and 2160 hours Given $A(1.5) = 0.4332$, $A(1.83) = 0.4664$, $A(2) = 0.4772$ **(07 Marks)**

c. Express Bernoulli distribution as exponential family form. **(07 Marks)**

Module-5

9 a. Find stationary points and indicate whether they are maximum, minimum or saddle points for the univariate function $f(x) = x^4 + 7x^3 + 5x^2 - 17x + 3$. **(06 Marks)**

b. Derive dual linear program using Lagrange duality for the linear program

$$\min_{x \in R^2} -\begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{subject to} \quad \begin{bmatrix} 2 & 2 \\ 2 & -4 \\ -2 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix}$$ **(07 Marks)**

c. For the sum of the loses $\ell(t)$ where $\ell : R \to R$ derive the converse conjugate. **(07 Marks)**

OR

10 a. Derive the dual quadratic program using Lagrange duality for the quadratic program.

$$\min_{x \in R^d} \frac{1}{2} x^T Q x + C^T x \quad \text{subject to } Ax \le b \text{ where } A \in R^{m \times d},\ b \in R^m \text{ and } C \in R^d.$$ **(10 Marks)**

b. Find the convex conjugate of a quadratic function $f(y) = \frac{\lambda}{2} yTk^{-1}y$ where $k \in R^{n \times n}$ is a positive definite matrix and $y \in R^n$. **(10 Marks)**

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